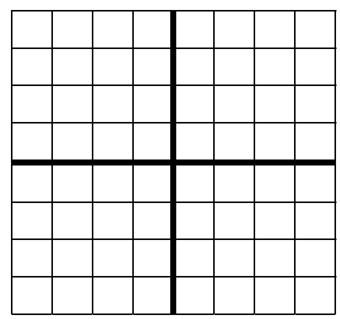
Graphing Workshop

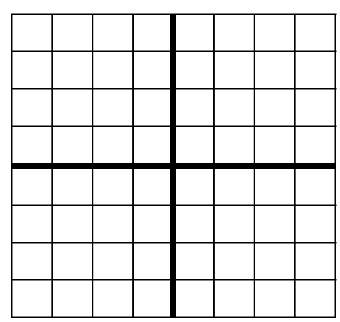
Determining the Slope of a Line

- Slope = $m = \frac{rise}{run} = \frac{\Delta y}{\Delta x} = \frac{y_1 - y_2}{x_1 - x_2}$, for **any** two points $(x_1, y_1), (x_2, y_2)$ on the line

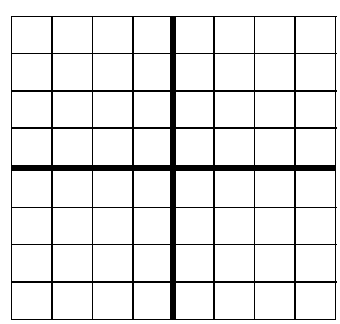
  - **Two Methods for Calculating the Slope of a Line:**
    - **Method 1: Counting Method**; Using the Formula: $m = \frac{rise}{run}$
      - *Exercise 1:* For each of the examples below, plot the given points and determine the slope of the line which passes through the given points:
        - (2, 1) and (-4, -3)



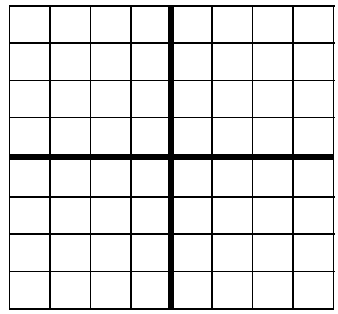
        - (1, 0) and (4, 4)



        - (-3, 3) and (4, -3)



- ☐ (-1, 3) and (4, 2)



- ☐ **Method 2: Calculating Using the Formula:** $m = \frac{\Delta y}{\Delta x} = \frac{y_1 - y_2}{x_1 - x_2}$

  - ☐ *Exercise 2:* For each of the examples below, determine the slope of the line which passes through the given points:

    - ☐ (2, 1) and (-4, -3)

    - ☐ (1, 0) and (4, 4)

    - ☐ (-3, 3) and (4, -3)

    - ☐ (-1, 3) and (4, 2)

Notes:

- Lines with positive slopes vs. lines with negative slopes
- Lines with steep slopes vs. lines with shallow slopes
- The counting method, and calculating using the formula above will give the same answer
- The slope of a line is the same throughout the line; so you can calculate a line's unique slope using any two points
- Parallel lines have the same slope (and as a result never cross each other)

Graphing Lines Using the Slope-Intercept Method

- **Slope-Intercept Form of a Line:** $y = mx + b$
  where $m =$ the slope of the line, and $(0, b) =$ the *y*-intercept of the line

- *Exercise 3:* For each of the following equations of lines, determine the slope and *y*-intercept, and then graph the equation

  - $y = -5x + 3$
  - $y = -5x - 2$
  - Note: Two lines with the same slope are parallel (i.e. they will never intersect)
  - $y = 3x - 6$
  - $y = -\frac{1}{2}x - 6$

- Note: Two lines with the same y-intercept will intersect at that point

- *Exercise 4:* What will be the relationship of the given pairs of lines:

  - $y = -\frac{3}{4}x + 4$, and $y = -\frac{3}{4}x$

  - $y = -\frac{3}{4}x + 4$, and $y = x + 4$

- *Exercise 5:* Consider the line:

  - $y = x$

  - Notes:

    - Goes through (i.e. bisects) the origin because the y-intercept is (0, 0)

    - It bisects the origin at a 45 degree angle because $m = 1$

Determining Points of Intersection

(i.e. finding the solution of a system of linear equations)

- What does it mean to be a solution to a system?

- Review:

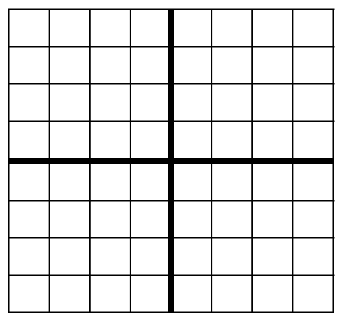
  - For two parallel lines there is no solution (i.e. it is an inconsistent system)

  - Lines with the same y-intercept have that point as it's solution

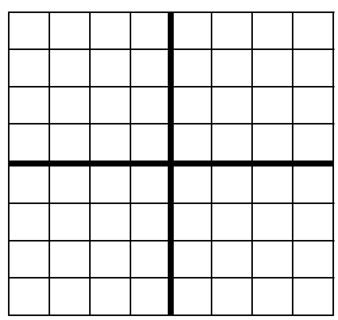
  - For the same line there are an infinite number of solutions (i.e. a dependent system)

    - Example: $x + y = 2$ and $y = -x + 2$

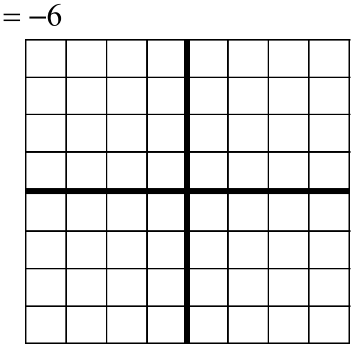
- **2 Methods for Determining the Solution to a System of Equations**
  - **Method 1: The Graphing Method**
    - Steps:
      - Carefully graph the first equation using the slope-intercept method
      - Carefully graph the second equation using the slope-intercept method
      - Determine where the lines intersect (that coordinate is the solution)
    - *Exercise 6:* Solve each of the systems below graphically:
      - $x + y = -2$ and $2x - 3y = -9$



      - $x - y = 2$ and $-3x + 3y = 6$



      - $x - y = 2$ and $-3x + 3y = -6$



- **Method 2: The Algebraic Method Using Substitution**
  - Steps:
    - Solve one of the equations for x or for *y*
    - Substitute your result into **the other** equation, and solve it (for the variable that remains)
    - Substitute your result into one of the original equations
    - Write your solution as a coordinate and check your solution
  - *Exercise 7*: Solve each of the systems below algebraically:
    - $x + y = -2$ and $2x - 3y = -9$
    - $x - y = 2$ and $-3x + 3y = 6$
    - $x - y = 2$ and $-3x + 3y = -6$