Problem Solving 101

Mathematics was developed in order to solve real world problems, so it seems only natural that we begin our course with a look at some problem solving techniques. Problem solving is more of an art than a science. It requires ingenuity, experience and a bag of tricks.

Perhaps the most important rule of problem solving is to be flexible. No one strategy works all of the time. Try to stay away from looking for the "one way" to solve a problem. There generally is more than one approach that can be used to solve a problem.

Having said this, most problem solving can be broken down into several basic steps. You should think of the steps outlined below as guidelines that will help you stay focused, rather than as a set of rules.

A Four Step Problem Solving Approach

Step 1: Understand the Problem
This seems obvious but is often missed. Be sure to read the problem until it is understood; in a math course this may require that you read each problem four, five or more times. Students will often start a problem that they do not understand, jotting down whatever comes to mind without having given the problem serious thought before hand. Determine where you are going, make sure you understand what the problem is asking. What assumptions can you make? Other things to try:

- Think about how this problem relates to other problems you may have come across, or other problems in the real world.
- Try listing the given information in the table or chart; look for a pattern.
- Draw a sketch to help visualize the problem.

Step 2: Make a Plan
Once you understand the problem think about how you might go about solving the problem. Often this can be the hardest step for students. Make a list of possible strategies and hints that might help in the overall solution of the problem. Remember to estimate what the answer should look like.

Step 3: Carry Out the Plan
This is the step where you actually work through the mathematical details of the problem. Work neatly and in an organized manner. All too often students work in a sloppy manner and then have a tremendous amount of difficulty going back over their work when looking for something to change or correct, or make errors due to sloppy penmanship. Be sure to write down each step and sketch the TI-83 viewscreen if applicable.

Step 4: Check and Interpret your Solution
This step is often overlooked. There is a tendency to feel that once we have a solution we are finished with the problem. However, the solution is meaningless if it is wrong or cannot be explained to others (remember we solve problems in the real world and often need to be able to explain our solutions to others). Perform a formal check algebraically or graphically if possible. Make sure your answer is reasonable (see below). Does your answer agree with the estimate you made earlier?

Always ask yourself: DOES MY ANSWER MAKE SENSE?
Many times we can catch a mistake if we take the time to think about the reasonableness of our answer.
If we are trying to determine how many miles our car can go on one gallon of gas, is 300 miles per gallon reasonable? If we are a nursing student and are determining dosage, is 50 aspirin reasonable to give a patient? These are just two examples of incorrect answers I have actually received from students who could have caught their mistakes if they had asked themselves if their answer made sense. Don't just stop when you get an answer -- think about that answer for a minute of so. It is 60 seconds of time well spent.

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If you have always had difficulty solving problems in the past the steps outlined above should help you organize your problem solving approach. But the only way to truly improve your ability to solve problems is through practice and experience. The more you problems you solve, the less daunting they will seem.

Additional Hints about Problem Solving
There may be more than one right answer How do we stop the greenhouse effect? There is no single best answer to this question. In mathematics $x = 4$ and $x = -4$ are both solutions to the problem $x^2 = 16$. Without additional information we have no way to determine whether both solutions are correct or if only one answer will solve the problem. If this were an area problem only one value for *x* would be accurate.
There may be more than one way to approach the problem Using the example above concerning the greenhouse effect - there will be more than one way to approach the problem.
Use appropriate tools Is a calculator an appropriate to solve a particular problem?
Use approximations Make problems easier by using approximations. Approximations are useful when checking a solution. If your exact answer is not even close to your approximate answer, then something is wrong.
Do not spin your wheels Do not get bogged down with a problem. If you are spending a lot of time on a problem and getting nowhere with it, try leaving the problem for a while. You will be amazed at what you might be able to do when you come back to the problem later on.

Some material adapted from Ellena Reda's "Introduction to Algebra".